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Predhovor

Vážení čitatelia,

štvrté tohtoročné číslo časopisu Studia Scientifica Facultatis Paedagogicae je monotematicky zamerané na problematiku týkajúcu sa ontogenézy geometrických schopností v kontexte procesu rozvíjania geometrického myslenia a uvažovania.

Vedecké bádanie v oblasti zameranej na mapovanie procesu vytvárania geometrických predstáv je spojené s vývinom chápania geometrických pojmov, ako mentálnych reprezentácií a schopnosti s nimi manipulovať. Aby sa v rámci matematického vzdelávania tvorili také mentálne schémy, ktorých výsledkom je korektné chápanie geometrického pojmu v celej jeho šírke, je potrebné rozvíjať nielen predstavy detí, ale aj spresňovať jazyk, rozvíjať reč, zabezpečiť dostatok skúseností a manipulačných činností, či vymedzovať množinu symbolov spájajúcich sa s príslušným pojmom. Ak v geometrickom vzdelávaní zanedbáme niektorú z dôležitých zložiek participujúcich na tvorbe mentálnych reprezentácií, môžu vzniknúť a stabilizovať sa také miskoncepcie, ktoré budú neprekonateľnou prekážkou v ďalšom matematickom vzdelávaní.

Predloženú publikáciu tvoria štúdie, ktoré reflektujú problematiku riešenú v rámci projektu VEGA 1/0440/15 s názvom *Geometrické koncepcie a miskoncepcie detí predškolského a školského veku*. Ambíciou riešiteľov projektu je prispieť k rozšíreniu a prehĺbeniu vedeckej teórie o hladinách geometrického myslenia detí aj v kontexte dosahu na pedagogickú prax. Preto autori publikovaných štúdií zamerali svoju pozornosť nielen na výskumnú zložku a interpretáciu výsledkov svojho výskumného pôsobenia v oblasti geometrie, ale aj na konkrétne návrhy overených didaktických prístupov v geometrii a jej vyučovaní. Hľadajú odpovede na koncepčné otázky súvisiace s rozsahom, obsahom a kvalitou geometrického vzdelávania na rôznych stupňoch škôl s prihliadnutím na požiadavky spoločenskej praxe o zvyšovaní kvality matematického vzdelávania súvisiaceho s konkrétnymi praktickými aplikáciami geometrie v rôznych študijných odboroch.

S potešením môžeme konštatovať, že téma o poznávacom procese v geometrii sa stretla so záujmom aj v zahraničí a publikácia má nadnárodný charakter, čo umožňuje aspoň parciálnu komparáciu súčasného stavu geometrického vzdelávania v medzinárodnom kontexte. Veríme, že publikácia poskytne námetovú platformu na hlbšie bádanie a prispeje k ucelenejším záverom v oblasti identifikácie významných atribútov, ktoré sa vzťahujú k téme rozvíjania geometrického myslenia.

Katarína Žilková

Mylné predstavy učiteľov predškolského veku v súvislosti s rovinnými útvarmi

Misconceptions of Primary Pre-Service Teachers Regarding Interior of 2D Shapes

Alenka Lipovec, Manja Podgoršek

Abstract

The aim of the research was to find out which component of a figural concept, visual or conceptual one, a primary pre-service teacher uses in solving tasks about basic geometrical concepts. We present the results of the research conducted on primary pre-service teachers (N=74) studying in Slovenia. Our results show that more than 70% of pre-service teachers had false or poor images of basic geometrical concepts and perceived boundary points only as common points in solving tasks. The results also show that the degree of unlimitedness is not the cause for mismatch of an evoked concept image and a formal definition, which raises questions for further research. Results were contrasted with former research regarding square as a figural concept. We argue that teaching geometry at primary level could be impeded by common content knowledge primary teachers' possess and urge to restructured teacher training programmes in the area of geometry education.

Keywords: teacher training, figural concept, geometry, mathematics education.

MESC: G10, G40

1. Van Hiele theory

The theory on the development of geometric thinking and concepts as well as in geometry didactics was set by the Dutch Dina van Hiele-Geldof and Pierre van Hiele in the 50's decade of the last century. Their initial theory assumes that an individual's development in the field of geometry follows a discreet hierarchical sequence of levels. These levels are numbered differently in different sources, starting with 0 or 1. Here we list the original characterization of the initial levels: level 0 - Figures are judged by appearance; level 1 - Figures are bearers of their properties; level 2 – Properties are ordered; level 3 – Thinking is concerned with the meaning of deduction, with the converse of a theorem, with axioms, with necessary and sufficient conditions. (van Hiele, 1984, p. 245). In their later works it is possible to find higher levels, but they are not significant for initial geometry training.

In van Hiele's theory there is a natural sequence of levels, which is partly independent of the teaching methods and that this achievement of levels is not biologically conditioned. The result of observation and thinking on the previous level becomes the object of manipulation on the next one. If the result of level 0 are classes of shapes (such as squares, triangles, ...), they become the object of research at level 1, where it comes to find the properties of each particular class object. Such a relationship between the levels prevents skipping them. Each level is characterized by its own vocabulary, which means that (good) communication between people who operate at different levels is made impossible. This is exactly what is important in the process of education. If a teacher keeps teaching geometry at a van Hiele level, which students do not attain and do not know the vocabulary for, symbols and relations between objects, such teaching is ineffective, because students do not know what the teacher is saying. Similarly, a teacher who expects answers at a van Hiele level, which is different from the student's one, cannot make sense of the students' responses (van Hiele, 1984).

Some subsequent studies have shown that achieving van Hiele levels also depends on the content and that it is not necessary for an individual to achieve the same (global) level for different contents (Wu & Ma, 2006). It may also be that the individual in reasoning or finding solutions simultaneously uses two or more consecutive levels, what probably depends on the complexity of the problem being solved. Here, the degree of a lower van Hiele level is more complete and remains an important part of a student's cognitive schemata (Matos, 1999, p. 183). A higher degree of acquisition of a lower van Hiele level means a greater certainty of operating at this level and this can lead to reasoning at a lower level.

2. Figural concept

Fischbein (1993) devoted his research to special nature the geometrical concept have. The amalgam of concept and a figure was named *figural concept* because of its double nature. Figural concepts simultaneously possess both conceptual and figural properties where an image is entirely controlled by a definition (Fischbein, 1993, p. 149). The ideal figural concept corresponds to the concept definition set by Tall and Vinner (1981), i. e. a formal definition adopted in a mathematical community. An individual's mental reflection of the ideal figural concept with all the connotations, ambiguities and uncertainties corresponds to the concept image of Tall and Vinner. The concept image represents individual's entire cognitive scheme of the concept (Fischbein, 1993, p. 150). The definition of the figural concept can therefore be included in the individual's concept image, yet only a part of a concept image is included in a current mental process of reasoning, and this evoked image does not necessarily include a formal definition.

3. Mathematical knowledge for teaching

As early as 1986, Shulman pointed out that the mere subject knowledge is not enough for teaching. Among other things, he introduced the notion of pedagogical content knowledge. This includes the knowledge of how a particular content is presented to students in an appropriate way. Later, Ball, Thames & Phelps (2008) adapted Shulman's model to the field of mathematics and introduced the concept of mathematical knowledge for teaching, which includes, on one hand, pedagogical content knowledge, and on the other hand, subject matter knowledge. Knowledge of the subject includes common content knowledge, which played an important role before Shulman and was the only one to be checked, and also specialized content knowledge and horizon content knowledge.

Several studies have confirmed that in problem solving a visual component of a concept is essential, even when the individual is familiar with a verbal concept definition, which may be inconsistent with the visual image. It is known that verbal learning of definitions does not suffice for the development of a concept (Matos, 1999). Students of different ages claimed that a point on the board and the one in the notebook are the same, because the point has no dimension and shape. The same students then claimed that the point of intersection of two lines is smaller than that of four lines (Fischbein, as cited in Fischbein, 1993). Similarly, students in other studies were able to explain what a triangle or an angle is, but they did not accept the shapes that were not consistent with their visual images although the shapes satisfied their requirements (e.g. Hershkowitz et. al., 1990; Matos, 1999; Clements & Sarama, 2009; Ozdemir Erdogan & Dur, 2014). Ward (2004) got similar results with pre-service teachers for the case of a (right-angled) triangle and a hexagon.

Bezgovšek Vodušek & Lipovec (2014) conducted a research, where primary pre-service teachers' concept image in the case of the square was analysed. The main emphasis was on a question whether participants perceived a square as hollow shape or as filled shape. Given the importance of a visual image, a problem may arise of distinguishing between the shape and the curve which bounds the shape. Results of this research show that only a small part of participants perceived the square as filled shape what would be in accordance with formal definition. The authors argued that participants image of a square as a frame totally dominated the conceptual part of the figural concepts square.

If a square is replaced by a disc / circle the situation is similar. Let us first explain what we mean when we write 1D-circle or 2D-circle. In Slovenian language two words are used. The word *krožnica* denote the boundary line of a disk / circle. The word *krog* denote the two dimensional shape bounded by that line. We will therefore write 1D-circle when in Slovenian language word *krožnica* is used and 2D-circle when two dimensional shape bounded by 1D-circle denoted as *krog* is used.

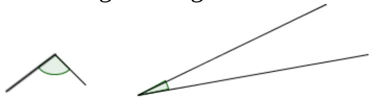
Our research is aimed at determining Slovenian pre-service teachers' conceptual and visual component of other basic geometric concepts 2D-circle, polygon and an angle and comparing that results to the results a concerning square.

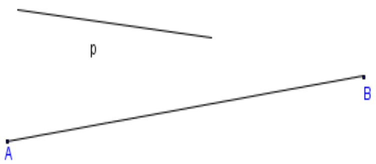
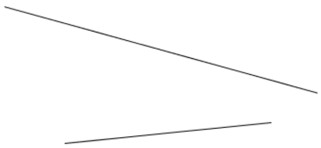
4. Methods

The research was conducted in February 2012 at the Faculty of Education in Maribor, Slovenia. It included 74 students of the second year of the Primary Education Program. Prior to entering the faculty, the students had finished elementary mathematics and four years of compulsory secondary-school mathematics (560 hours). In many European countries students can choose the level of mathematics in a high school and thus the number of hours, however, this is not possible in Slovenia. This means that the participants in this study had more hours of mathematics compared to their colleagues from other EU countries. TIMSS advanced 2008 showed good mathematical knowledge of 40 percent of the total Slovenian population (Slovenian students achieved average results). It would therefore be expected that they have a good common content knowledge. The study was based on the descriptive, causal and non-experimental methods of pedagogical research. It enables us to explore the role of the conceptual and the visual component of a figural concept among primary pre-service teachers when solving school geometrical tasks.

In order to determine which component of a figural concept, a visual or a conceptual one, is predominant with pre-service teachers in solving tasks in the field of basic geometrical concepts, we used a written test. The participants were solving the test individually during regular tutorials in mathematics. Tasks involving relations (especially in terms of common points) between objects in the same plane were used to find out whether pre-service teachers in solving them also take into account the properties not included in the visual representation (for example limitlessness of a straight line, the interior of an angle). To illustrate tasks, we provide them in table 1.

Table 1. Presentation of tasks.

Task	**Task**
A	Which angle is larger?  Figure 1 Figure 2 a) Angle in Fig. 1. b) Angle in Fig. 2.

Task	**Task**
pAB	Which is longer?  a) AB b) p
pq	Do objects on a figure intersect each other?  a) yes b) no
pΠ1	Draw a straight line and a plane which have exactly 1 common point.
pΠ2	Draw a straight line and a plane which have exactly 2 common points.
cSQ2	Draw a 1D-circle and a square so that they will have exactly 2 common points.
cSQ8	Draw a 1D-circle and a square so that they will have exactly 8 common points.
cT2	Draw a 1D-circle and a triangle so that they will have exactly 2 common points.
cT6	Draw a 1D-circle and a triangle so that they will have exactly 6 common points.
pA1	Draw a line and an angle which have exactly 1 common point.
pA2	Draw a line and an angle which have exactly 2 common points.

**The typical convention for drawing and labelling lines and segments in Slovenia is as represented in tasks pAB and pq.*

The tasks were selected so that the usual visual representation of concepts in the task without considering the conceptual component lead to a wrong solution. We then compared the percentage of correct solutions of tasks depending on what concepts they included. We considered two aspects, the topological dimension and the degree of boundedness. With respect to the first aspect, the included concepts were one-dimensional (1D) or two-dimensional (2D). The degree of boundedness should be understood as the extent to which the concept is limited, i. e. bounded, partly bounded or unbounded. Table 2 shows examples of concepts with varying degrees of boundedness. It seems natural to expect a poorer performance in tasks involving several unlimited objects. In the open type tasks, we then analysed wrong answers in more detail.

Table 2. Examples of concepts with respect to the degree of boundedness.

	Degree of boundedness		
	Bounded	Partly bounded	Unbounded
1D	1D-circle	ray	straight line
2D	square, triangle	angle	plane

5. Results

Table 3 presents the results for each individual task. The answers of pre-service teachers in solving the tasks which required drawing the solution can be classified into the following categories: correct answer, incorrect answers, where we distinguished answers with only boundary points, no answer.

Table 3. Preservice teachers' students' solutions regarding correctness.

Task	Correct		Incorrect				No answer	
			Only boundary points		Other			
	f	f %	f	f %	f	f %	f	f %
A	72	97,3 %	62	83.8%	0	0.0%	2	2.7%
pAB	62	83,8%	/	/	12	16,2%	0	0.0%
pq	43	58,1%	/	/	31	41,9%	0	0.0%
pΠ1	0	0.0%	70	94.6%	0	0.0%	4	5.4%
pΠ2	39	52.7%	29	39.2%	2	2.7%	4	5.4%
cSQ2	0	0.0%	64	86.5%	5	6.8%	5	6.8%
cSQ8	30	40.5%	33	44.6%	8	10.8%	3	4.1%
cT2	1	1.4%	51	68.9%	15	20.3%	7	9.5%

The results show that some tasks were solved very successfully, where other tasks were solved completely wrong. Among incorrect answers the vast majority (72,8 %) of respondents pointed out only boundary points as intersection points. It seems they perceive a shape only as boundaries without interior.

The result is in accordance with research Bezgovšek Vodušek & Lipovec (2014) conducted. All two dimensional shapes were perceived without boundaries. The same observation could be made for concept of an angle.

We will now compare the performance when increasing unboundedness in the tasks which included concepts of equal dimensions (table 4). We can see that by increasing unboundedness, performance decreases in solving tasks with two D1 objects (pAB - 83.8% and pq - 58.1%), but not in the case of one D1 object and one D2 object (cSQ2 – 13.5 %, cT2 – 9.5 %; pA1 – 52.7 % in pΠ1 – 40.5 %). It can be concluded from this that performance is not directly related to the degree of limitedness. A general low performance mainly points to frequent inconsistencies between definitions of figural concepts and their concept images shown by respondents.

Table 4. Correct answers with respect to boundedness and the dimension of included concepts.

		Limited			Partly limited			Unlimited					
		D2			D2			D1			D2		
		Task**	f	f %	Task	f	f %	Task	f	f %	Task	f	f %
Limited	D1	cSQ2	10	13.5%				pAB	62	83.8 %			
		cSQ8	*0* *15%**	*0.0 %* *20.3%**									
		cT2	7	9.5%									
		cT6	*0* *4**	*0.0 %* *5.4 %**									
Partly limited	D2				A	72	97.3%	pA1	39	52.7 %			
								pA2	*0* *4**	*0.0 %* *5.4 %**			
Unlimited	D1							pq	43	58.1%	pП1	30	40.5%
											pП2	*1* *7**	*1.4%* *9.5 %**

** Tasks are shown in Table 1. * No answer.

6. Discussion

Teachers' content pedagogical knowledge is related to teachers' common content knowledge and specialized content knowledge. The results of the research show that in solving complex tasks, the individual's visual representation of the concept has priority therefore pre-service teachers' common content knowledge could be perceived as weak. The results of the research show that the inconsistencies between the definition and the used part of the individual's concept image do not increase with the intensification of unboundedness of the concept and the concept topological dimension. In solving complex tasks, the individual's visual representation of the concept has priority. The highest percentage of correct answers shows the task (A), which is a routine task in the Slovenian curriculum for primary and secondary schools. In fact, both curriculums put great emphasis on measuring angles and constructing angles of a certain size. The same is true in case of a straight line. The performance is very high in the task including a straight line and a line segment (pAB), which in fact requires knowledge of the verbal definition of a straight line (a finite line versus an infinite one). Success, however, drops significantly in the task pq, although it seems that the knowledge sufficient to successfully solve the tasks pAB, also suffices to successfully solve the task pq. However, in the task pq it is not enough to merely word the properties of a straight line, but it is necessary to consider the properties which are not included in the visual presentation.

Our results also show that more than 70 % of pre-service teachers had false or poor images of basic geometrical concepts and perceived boundary points only as common points in solving tasks. This confirms the above mentioned priority of visual presentations. The percentage of responses in the category "only boundary points" is quite high (72,8 %), but lower that was in the case of a square where more than 90 % of participants perceive square as a frame. Attaining higher Van Hiele levels in the context of teaching is a pre-requisite for in-depth knowledge of geometry, which falls within the scope of specialized content knowledge. Based on the results, we may assume that elementary pre-service teachers did not achieve higher Van Hiele levels. Their thinking is mostly limited to the visual image of a concept and rarely includes conceptual elements of this figural concept. This way of thinking is characteristic for the first two Van Hiele levels. Prospective teachers therefore do not reach the level expected by the curriculum for students who will be taught by them. The results are consistent with the results of other studies (Pandiscio & Knight, 2010).

7. Conclusion

Geometry has a great importance for individuals to develop their problem-solving, critical thinking, reasoning and higher-order thinking skills (NCTM, 2000). For this reason teaching of geometrical concepts has an important place in meaningful learning. According to a theory of a figural concept, the highest level of the geometrical reasoning consists of providing students with tools that can help them in building the interaction between the concepts and the image. Teachers should therefore be equipped with strong common content knowledge and specialized content knowledge. Based on our results we argue that common content knowledge of pre-service teachers, which is the foundation of mathematical knowledge for teaching, especially in teaching an angle and a two-dimensional shape is insufficient. Additional research will be necessary to determine whether: a) pre-service teachers comprehend the mentioned geometrical concepts purely as their boundary, b) they comprehend the point as the intersection of lines only, and c) they are unable to see the interior of a concept as a (dense) set of points. By knowing this, we will discover misconceptions, which have not yet been addressed. If these findings help us develop effective teaching of (pre-service) teachers, which will enrich currently poor but key visual images of geometric concepts, we will contribute to raising the level of knowledge of geometry also with students.

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